

Exercise 17

Prove that $\frac{d}{dx}(\csc x) = -\csc x \cot x$.

Solution

Use the quotient rule to differentiate the function.

$$\begin{aligned}\frac{d}{dx}(\csc x) &= \frac{d}{dx} \left(\frac{1}{\sin x} \right) \\ &= \frac{\left[\frac{d}{dx}(1) \right] \sin x - 1 \left[\frac{d}{dx}(\sin x) \right]}{(\sin x)^2} \\ &= \frac{(0) \sin x - 1(\cos x)}{\sin^2 x} \\ &= \frac{-\cos x}{\sin^2 x} \\ &= -\frac{1}{\sin x} \frac{\cos x}{\sin x} \\ &= -\csc x \cot x\end{aligned}$$